An inner-outer iterations approach based on the Golub-Kahan bidiagonalization method applied to saddle point problems

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Abstract

Saddle point problems arise in many disciplines and applications, such as constrained optimisation or mixed finite elements, and the literature about solution techniques is vast. In our application, we look at the structural analysis of the containment buildings of nuclear power plants. For this clearly critical industrial application, it is important to have a reliable and efficient simulation software, such as Code_Aster developed by EDF (Electricité De France). The example of the containment building is a problem in linear elasticity, wherein one-, two- and three-dimensional finite elements (representing the outer shell, metallic wires and the concrete block) are coupled by multi-point constraints (MPCs). The MPCs are enforced by Lagrange multipliers and thus the resulting matrix to solve is indefinite, and of a 2x2 block structure. Clearly, the complexity of the structure of the building is reflected in the size of the stiffness as well as constraint matrix, and is rather large. Classic iterative solvers do not give satisfactory results, which explains the need for new scalable iterative solvers for this problem class.

With this as our motivation, we will present how a variant of the Golub-Kahan bidiagonalization algorithm, which has been widely used in solving least-squares problems and in the computation of the singular value decomposition of rectangular matrices [1], can be used as an iterative solver for indefinite matrices described above. Let now M be the positive-definite (1,1)-block (e.g. the elasticity stiffness matrix) of the augmented system. For each iteration of the generalized GKB method, a linear system Mz=b has to be solved. Our algorithm needs, with a good choice of a parameter and a direct solver for the inner system, only few iterations to converge and the number of iterations is independent of the problem size of the finite element discretisation. For complex applications, the matrix M is however large, and the solution step therefore requires an iterative solver. The performance of this inner-outer iterative GKB method, in terms of accuracy and computational cost, depends clearly on the inner iterative solver. In this talk, we will compare choices for solvers and preconditioners, and show numerical results for a test case obtained from EDF and the Stokes equation.